An Algorithm for Magnetically Dumping GPS Satellite Angular Momentum

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This paper discusses an algorithm for a ground-controlled method which dumps unwanted satellite angular momentum, using the interaction between the Earth's magnetic field and the Global Positioning System (GPS) satellite's two electromagnets. This method eliminates satellite ephemeris errors produced by reaction control system (RCS) momentum dumping. First, the algorithm chooses time intervals for magnet operation which enable the two magnets to dump about all three axes and insures the interval endpoints occur when the satellite can communicate with a ground station. Then the algorithm calculates the required settings of magnet strengths for each interval using the technique of minimum normalization, and insures that calculated values do not exceed magnet capabilities. Finally, the algorithm reduces the number of magnet-setting commands if possible. Computer simulations indicate that this method can magnetically dump any expected satellite momentum using existing GPS systems without introducing ephemeris errors.

Nomenclature

A	= array of influence coefficients
B	= Earth magnetic field vector
h	= satellite unwanted angular momentum component
H	= satellite unwanted angular momentum vector
K_p	= magnetic field disturbance parameter
m	= satellite magnet values
M	= vector of x and y magnet values at various times
r	= radius of the satellite's orbit
R_e	= radius of the Earth
T	= satellite torque vector
x,y,z	= satellite axes (Fig. 1)
Δ	= numerical difference between endpoints

Subscripts

ℓ	= limited values
u	= undetermined values
x,y,z	= satellite axes (Fig. 1)
$1,2,3$	= 1st, 2nd, 3rd time interval

Introduction

THE Global Positioning System (GPS) is a worldwide navigation frame eventually providing users with a three-dimensional location with accuracies of ± 10 m. The extraterrestial portion of GPS consists of 24 attitude-controlled, Earth-pointing satellites, each in a 63 deg inclined circular orbit with a half-day period. The system requires precise satellite ephemerides to achieve the desired navigation accuracies. A major source of ephemeris error is the expulsion of unwanted angular momentum which may occur as often as every second or third day. Torques caused primarily by the satellite's magnetic field and by solar pressure produce angular momentum which accumulates on onboard momentum wheels. Each wheel's momentum must be discarded before the wheel speed becomes excessive. Using the reaction control system thrusters to dump angular momentum produces a net force (due to thruster misalignments and imbalances) and results in velocity changes (ΔV's) which introduce ephemeris errors.

A previous study demonstrated the feasibility of dumping GPS satellite angular momentum magnetically, using the interaction between the Earth's magnetic field and the satellite's onboard electromagnets, a technique which eliminates this source of ΔV's and its associated ephemeris errors.[1] That study recommended that future GPS satellites use an autonomous magnetic control scheme, and that present satellites employ some interim method using current GPS onboard and ground-support equipment. This report presents such an interim method.

Magnetically controlled and stabilized satellites are not new.[2,3] However, the GPS presents several problems: 1) the satellite has no magnetometer, requiring calculation rather than measurement of the Earth's field strength; 2) it has electromagnets only on its x and y axes, posing special problems for dumping about certain axes; 3) the strengths of the electromagnets are limited to values between ± 7500 pole-cm, restricting torques available in the relatively weak Earth magnetic field at the satellite altitude of 3.17 R_e; and 4) the satellite has no command storage capability, requiring line-of-sight communication with a ground station to change magnet settings.

The following sections describe an algorithm which overcomes these problems using present GPS systems and shows the results of that algorithm in computer simulations.

Assumptions

To contend with operational problems, the algorithm makes the following assumptions. The magnet values can be set whenever the satellite is at least 7 deg above the horizon of an operative ground station. (Provisions are made for any one of the six ground control stations being inoperative.) Four switches per satellite per day will not put execessive strain on the operational ground-support equipment (SCF). The satellite's attitude control system orients the satellite to the accuracy required by the GPS specifications.

The maximum momentum buildup during one day is not expected to exceed 0.046 N·m·s in roll and 0.065 N·m·s in pitch. This assumes a residual magnetism of 3000 pole-cm on each spacecraft axis reduced by the redundant roll magnet to 1000 pole-cm on the x axis. In addition, there is a 2000 pole-cm dipole due to the solar panel current loop and a 1 deg solar panel misalignment.

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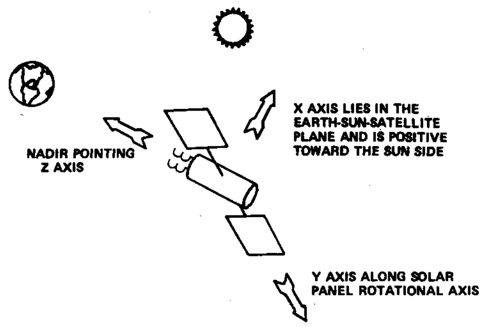

Fig. 1 GPS spacecraft axes orientations.

Determining Time Intervals

The interaction between the satellite's magnetic field and the Earth's magnetic field produces a torque:

$$T = M \times B \tag{1}$$

Acting over a time interval, this torque changes the satellite's angular momentum.

The direction of the average torque over that time interval depends on the satellite's orientation to the Earth's magnetic field. Over two different time intervals, the average torques produced by the same satellite magnet will generally be non-colinear because of the dynamic relationship between the satellite's magnetic field and that of the Earth. Therefore, dumping angular momentum about three axes can be accomplished with a minimum of three magnet values over three time intervals.

The algorithm chooses time intervals which produce large momentum changes about a single satellite axis and small changes about the other two axes. Figures 2 and 3 illustrate characteristic cyclic behavior of angular momentum due to constant magnet settings initiated at the ascending node. Some of the greatest momentum changes about a single spacecraft axis occur over anomaly intervals of 90 deg between the nodes and the antinodes. Thus, these points are candidates for the time-interval endpoints.

Figure 4 shows the coverage area for present GPS ground stations. There will always be a ground station within line-of-sight if switches occur above the northern hemisphere. If the only ground station visible is nonoperational, that switching time moves to the nearest time when an operational site is visible. Finally, it is desirable to have all the magnet switches occur during one 12-h orbit, which gives two opportunities per day to achieve a momentum dump. For the preceding reasons, the magnet switching times are normally chosen at arguments of latitude of 0, 90, 180, and 360 deg.

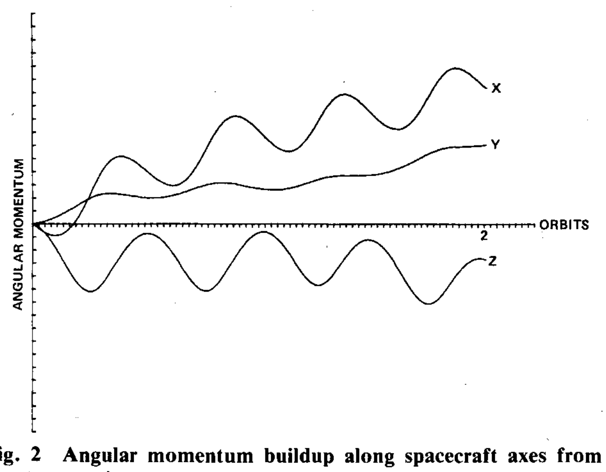

Fig. 2 Angular momentum buildup along spacecraft axes from a constant x-axis magnet.

Determining Magnet Values

After the algorithm chooses the time intervals, it determines magnet values which, acting over those time intervals, will dump the unwanted angular momentum. Although the problem of determining the magnet values is in the classical form of a linear programming problem, a more efficient method of solution is the minimum normalization solution to an underspecified matrix equation. Since each of the magnets affects the angular momentum independently over the time intervals, an expression for the momentum caused by the magnets is

$$H = [A]M \tag{2}$$

where

$$[A] \equiv \begin{bmatrix} \left.\frac{\Delta h_x}{\Delta m_x}\right|_1 & \left.\frac{\Delta h_x}{\Delta m_y}\right|_1 & \left.\frac{\Delta h_x}{\Delta m_x}\right|_2 & \left.\frac{\Delta h_x}{\Delta m_y}\right|_2 & \left.\frac{\Delta h_x}{\Delta m_x}\right|_3 & \left.\frac{\Delta h_x}{\Delta m_y}\right|_3 \\ \left.\frac{\Delta h_y}{\Delta m_x}\right|_1 & \left.\frac{\Delta h_y}{\Delta m_y}\right|_1 & \left.\frac{\Delta h_y}{\Delta m_x}\right|_2 & \left.\frac{\Delta h_y}{\Delta m_y}\right|_2 & \left.\frac{\Delta h_y}{\Delta m_x}\right|_3 & \left.\frac{\Delta h_y}{\Delta m_y}\right|_3 \\ \left.\frac{\Delta h_z}{\Delta m_x}\right|_1 & \left.\frac{\Delta h_z}{\Delta m_y}\right|_1 & \left.\frac{\Delta h_z}{\Delta m_x}\right|_2 & \left.\frac{\Delta h_z}{\Delta m_y}\right|_2 & \left.\frac{\Delta h_z}{\Delta m_x}\right|_3 & \left.\frac{\Delta h_z}{\Delta m_y}\right|_3 \end{bmatrix} \tag{3}$$

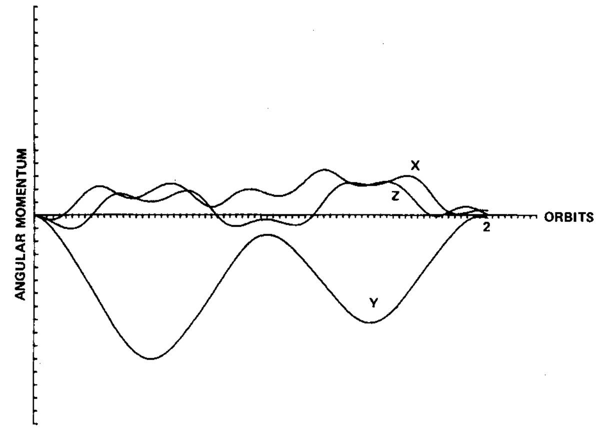

Fig. 3 Angular momentum buildup along spacecraft axes from a constant y-axis magnet.

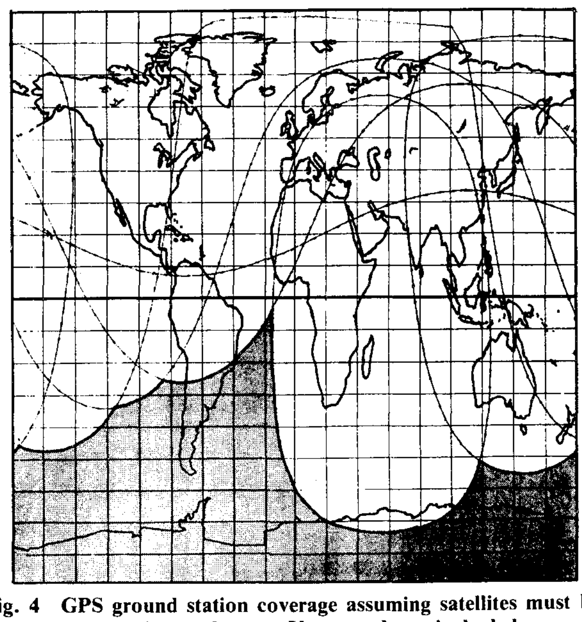

Fig. 4 GPS ground station coverage assuming satellites must be 7 deg above the horizon to be seen. Uncovered area is shaded.

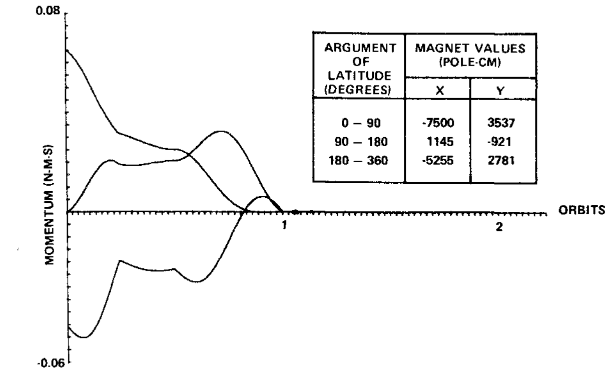

Fig. 5 Dump of (−0.046, 0.065, 0.0) N·m·s momentum, 0000 h, June 22, 1977.

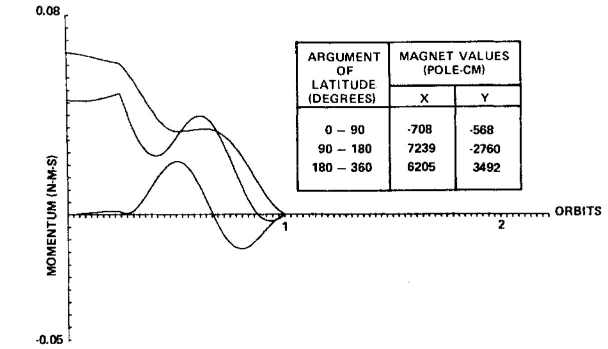

Fig. 6 Dump of (0.046, 0.065, 0.0) N·m·s momentum, 0000 h, June 22, 1977.

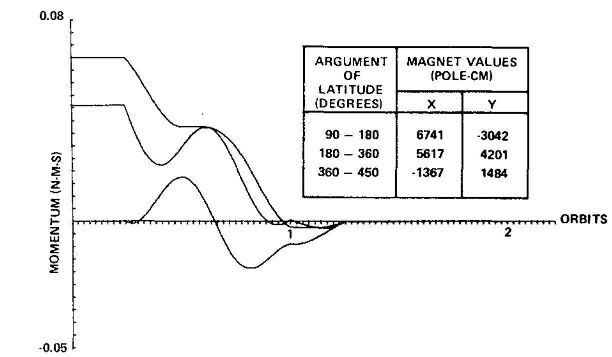

Fig. 7 Dump of (0.046, 0.065, 0.0) N·m·s momentum, 0000 h, June 22, 1977.

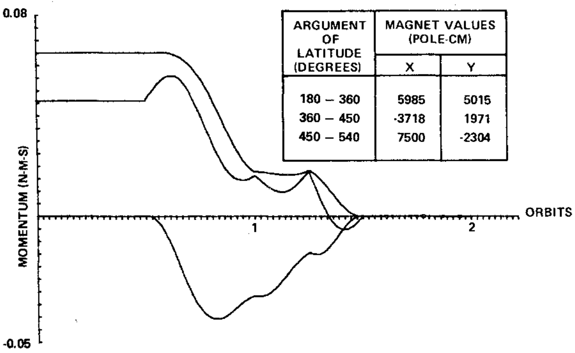

Fig. 8 Dump of (0.046, 0.065, 0.0) N·m·s momentum, 0000 h, June 22, 1977.

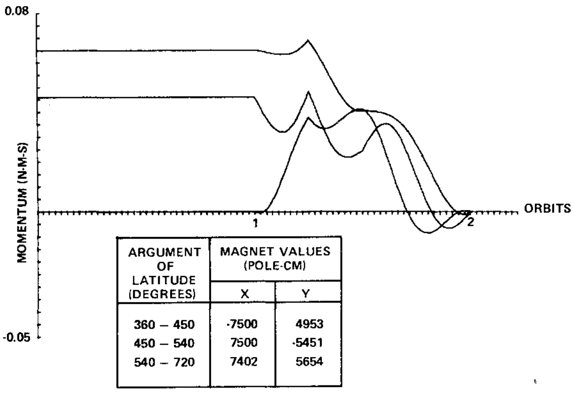

Fig. 9 Dump of (0.046, 0.065, 0.0) N·m·s momentum, 0000 h, June 22, 1977.

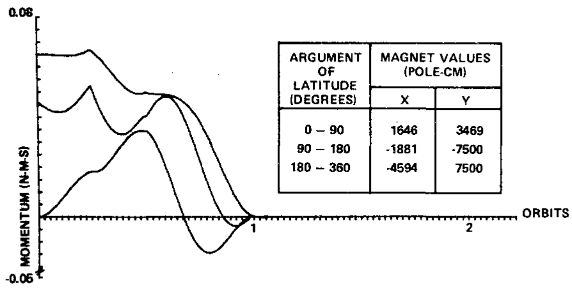

Fig. 10 Dump of (0.046), 0.065, 0.0) N·m·s momentum, 0000 h, March 22, 1977.

where

$$M=\begin{bmatrix} m_{x1} \\ m_{y1} \\ m_{x2} \\ m_{y2} \\ m_{x3} \\ m_{y3} \end{bmatrix}, \tag{4}$$

$$H=\begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} \tag{5}$$

The minimum normalization solution for the magnet values is

$$M=[A]^T([A][A]^T)^{-1}H \tag{6}$$

After calculating the magnet values, the algorithm attempts to reduce the number of required magnet settings by eliminating the extreme time interval with the smaller magnet magnitude, and then solving for new magnet values using an equation similar to Eq. (6) but of reduced dimension. If all the new magnet values are realizable, the algorithm is finished.

Otherwise, whenever any magnet values fall outside the range of ±7500 pole-cm, the value furthest outside the range is limited to either positive or negative 7500 pole-cm and Eq. (2) is rearranged and partitioned to isolate the limited value:

$$H=[A_u \vdots A_\ell]\begin{bmatrix} M_u \\ \cdots\cdots \\ m_\ell \end{bmatrix} \tag{7}$$

The minimum normalization solution for the unlimited magnet value is

$$M_u=[A_u]^T([A_u][A_u]^T)^{-1}(H-[A_\ell]m_\ell) \tag{8}$$

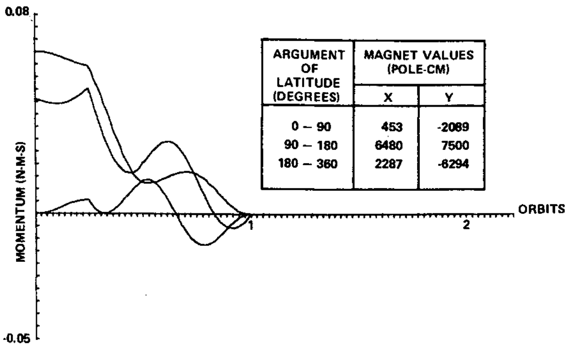

Fig. 11 Dump of (0.046, 0.065, 0.0) N·m·s momentum, 0000 h, September 21, 1977.

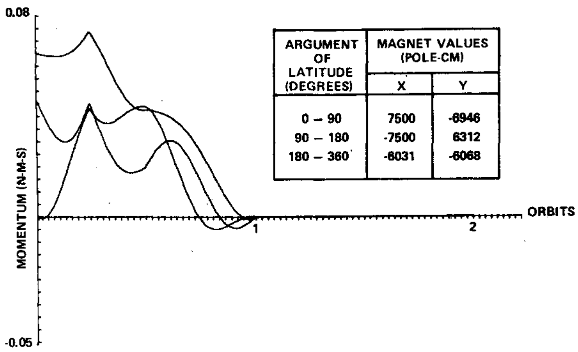

Fig. 12 Dump of (0.046, 0.065, 0.0) N·m·s momentum, 0000 h, December 21, 1977.

If any of the new magnet values are outside the range of realizable values, then the worst one is limited and isolated and the process is repeated for a decreased number of undetermined magnet values.

When the number of undetermined values reaches two, there are two possibilities: 1) the algorithm returns to the unreduced system (with all three time intervals) and repeats the process; and 2) the algorithm is already working on the unreduced system, it then switches to the least-squares solution

$$M_u = ([A_u]^T [A_u])^{-1} [A_u]^T (\bar{H} - [A_\ell] \bar{M}_\ell) \tag{9}$$

The process of limiting unrealizable values continues until it either reaches a feasible solution or all the values have been limited.

In all simulations studied, the anticipated worst-case momentum buildup is completely dumped with no more than four switches.

Figure 5 shows the angular momentum about each of the spacecraft axes during a typical magnetic dump, assuming no other external torques are acting. The momentum buildup due to magnetism is a linear process, so a buildup of (– 0.046, 0.065, 0.0) N·m·s is equivalent to (0.046, – 0.065, 0.0) N·m·s. The other worst-case situation, (0.046, 0.065, 0.0) N·m·s dump, is presented in Fig. 6. [The magnet values for a dump of (– 0.046, – 0.065, 0.0) N·m·s are the negative of the values given in Fig. 6.]

Figures 7 – 9 show the same dump as Fig. 6 but with delayed initial switching times for each successive figure.

Figures 10 – 12 show the same amount of momentum being dumped on different days of the year (the Earth-Sun-satellite geometry is varied). On these days, the actual worst-case momentum buildup is not as difficult to dump as the (0.046, 0.065, 0.0) N·m·s case. Therefore, for better comparison, the absolute worst case is presented. Notice that more magnet values are set to the limit (± 7500 pole-cm) on these days than in the reference case (Fig. 6).

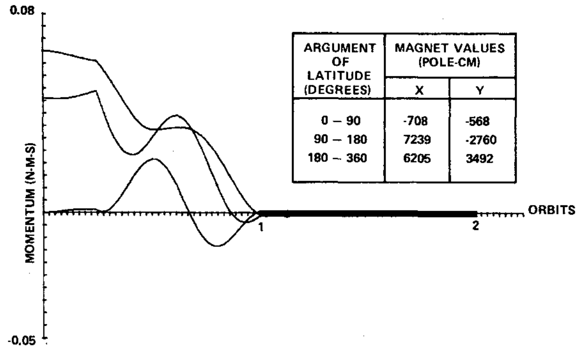

Fig. 13 Dump of (0.046, 0.065, 0.0) N·m·s momentum using quiet magnet values during disturbed period (see also Fig. 6).

During periods of solar activity (sunspots, solar flares, etc.), the Earth's magnetic field is disturbed for periods of several hours. Figure 13 illustrates the effects of a disturbed magnetic field on the dump. Residual momentum in this case is (– 0.00128, – 0.00082, 0.00061) N·m·s. The disturbance used is from the Mead-Fairfield model[4] for $K_p > 2$. This figure shows that expected field disturbances do not seriously affect the ability to dump momentum magnetically.

Conclusions

The figures clearly show that anticipated worst-case momentum can be dumped using present GPS satellite hardware, software, and associated ground electronics. The dumps require a maximum of four switches per satellite per day to eliminate the worst-case momentum buildup. Other cases indicate that in many instances the momentum can be dumped using only two time intervals.

Appendix: Magnetic Field Models

Quantitative geomagnetic field models are available primarily in two forms: those based on measurements at the Earth's surface,[5-8] and those based on extraterrestrial satellite magnetometer data.[4] Unfortunately, there is a region in space (between 4 and 5 R_e) for which neither model is designed to be used. The GPS orbit lies within this region ($r = 4.17 R_e$).

Previous analysis has shown that, for this application, the differences between using the two types of models is insignificant.[1] This study uses the Mead-Fairfield model based on satellite magnetometer data taken primarily beyond 6 Earth radii. There are two reasons for this choice: 1) this model provides an independent check on various other GPS analyses which use the quadrapole model; and 2) the Mead-Fairfield model incorporates an option to simulate a disturbed magnetic field. Since the GPS lifetime coincides with a period of maximum solar activity, the disturbed-field model allows a qualitative investigation of the effects of this activity.

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